Horizontal and vertical shifts

In this lesson, we're looking horizontal and vertical shifts. We'll start with sine and cosine functions in the form

$$\sin(\theta + c)$$

$$\cos(\theta + c)$$

where c is a nonzero real number. The graphs of these functions have a horizontal shift. In other words,

if c is positive,

then $\sin(\theta + c)$ is shifted c units to the left horizontally of $\sin\theta$,

and $\cos(\theta + c)$ is shifted c units to the left horizontally of $\cos\theta$.

if c is negative,

then $\sin(\theta + c)$ is shifted c units to the right horizontally of $\sin\theta$,

and $\cos(\theta + c)$ is shifted c units to the right horizontally of $\cos\theta$.

Example

Make a table of values of $\sin\theta$ and $\sin(\theta + (\pi/4))$ at multiples for θ of $\pi/4$, and then sketch the graphs of both functions.

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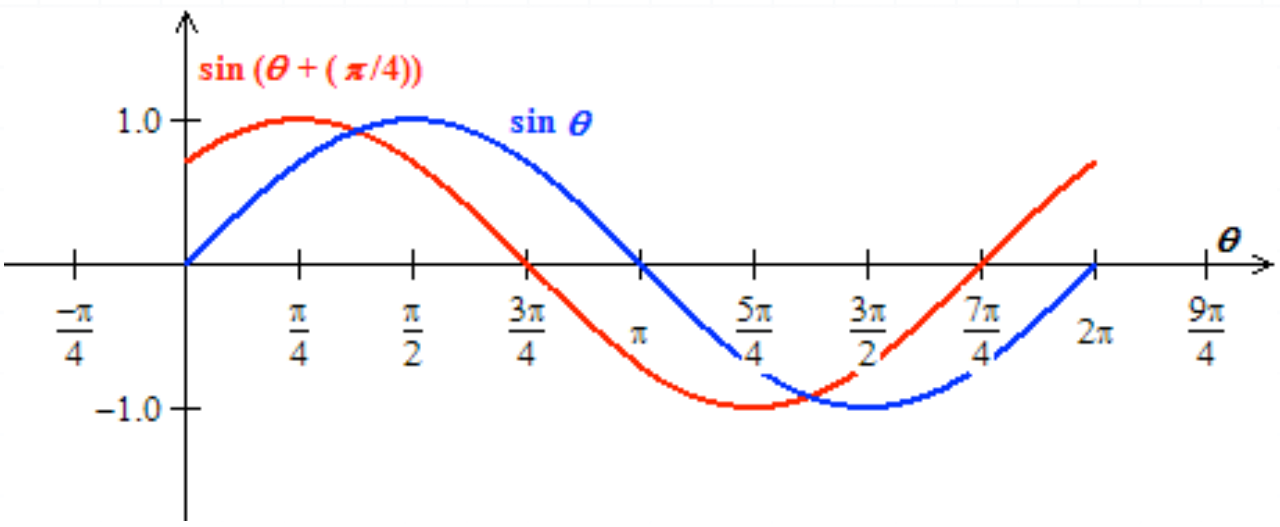
θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin\theta$	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$	0
$\sin\left(\theta+\frac{\pi}{4}\right)$	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$

Here, $c = \pi/4$ so $\sin(\theta + \pi/4)$ is shifted $\pi/4$ units to the left of $\sin\theta$.



Next, we'll consider a cosine function whose graph has a negative horizontal shift.

Example

Make a table of values of $\cos\theta$ and $\cos(\theta - (\pi/3))$ at multiples for θ of $\pi/6$, and then sketch the graphs of both functions.

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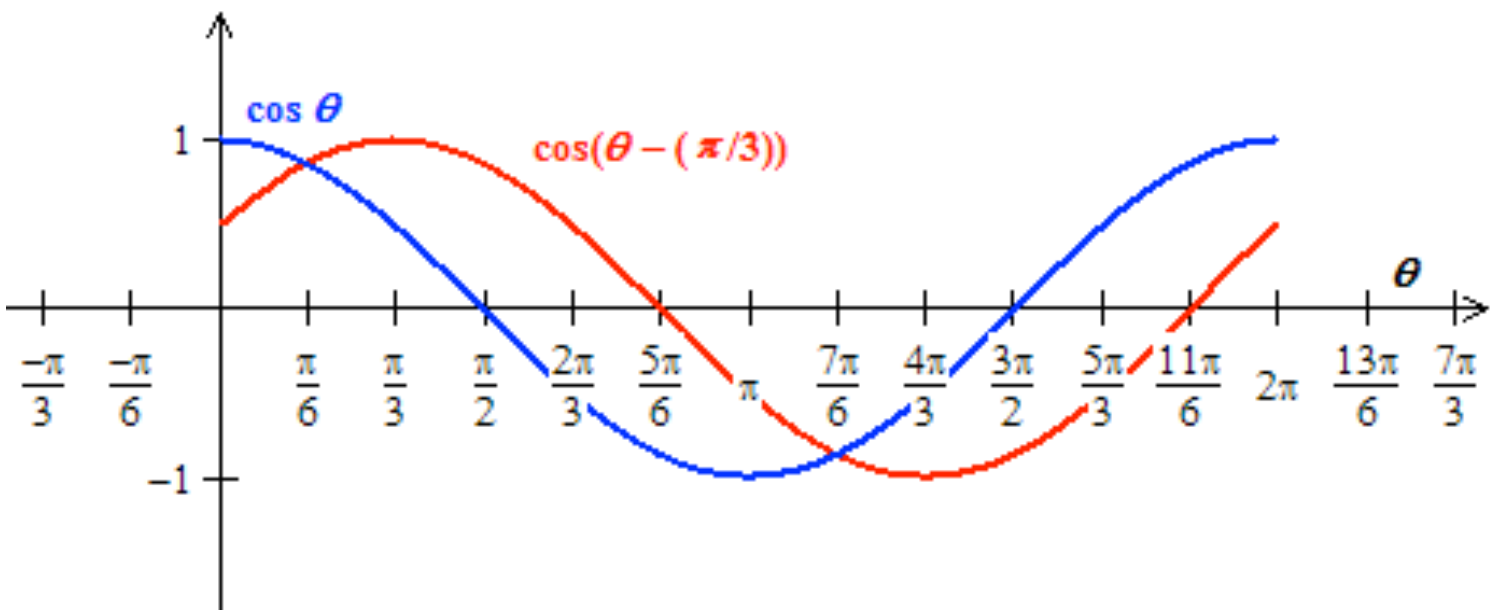
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\cos\left(\theta-\frac{\pi}{3}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$

and

θ	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\cos\theta$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\left(\theta-\frac{\pi}{3}\right)$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$

Since $c = -\pi/3$, $\cos(\theta-(\pi/3))$ is shifted $\pi/3$ units to the right of $\cos\theta$.



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Now let's look at functions in the form

$$\sin\theta + d$$

$$\cos\theta + d$$

where d is a nonzero real number. The graphs of these functions have a vertical shift. In other words,

if d is positive,

then $\sin\theta + d$ is shifted d units upward from $\sin\theta$,

and $\cos\theta + d$ is shifted d units upward from $\cos\theta$.

if d is negative,

then $\sin\theta + d$ is shifted d units downward from $\sin\theta$,

and $\cos\theta + d$ is shifted d units downward from $\cos\theta$.

The amplitude of a sine or cosine function with vertical shift, is one-half of the difference between its maximum and minimum values. The maximum value is $1 + d$ and the minimum value is $-1 + d$, so the amplitude of $\sin\theta + d$ or $\cos\theta + d$ is 1, because

$$\frac{1}{2}\left[(1+d) - (-1+d)\right]$$

$$\frac{1}{2}(1 + d + 1 - d)$$

$$\frac{1}{2}(2) = 1$$

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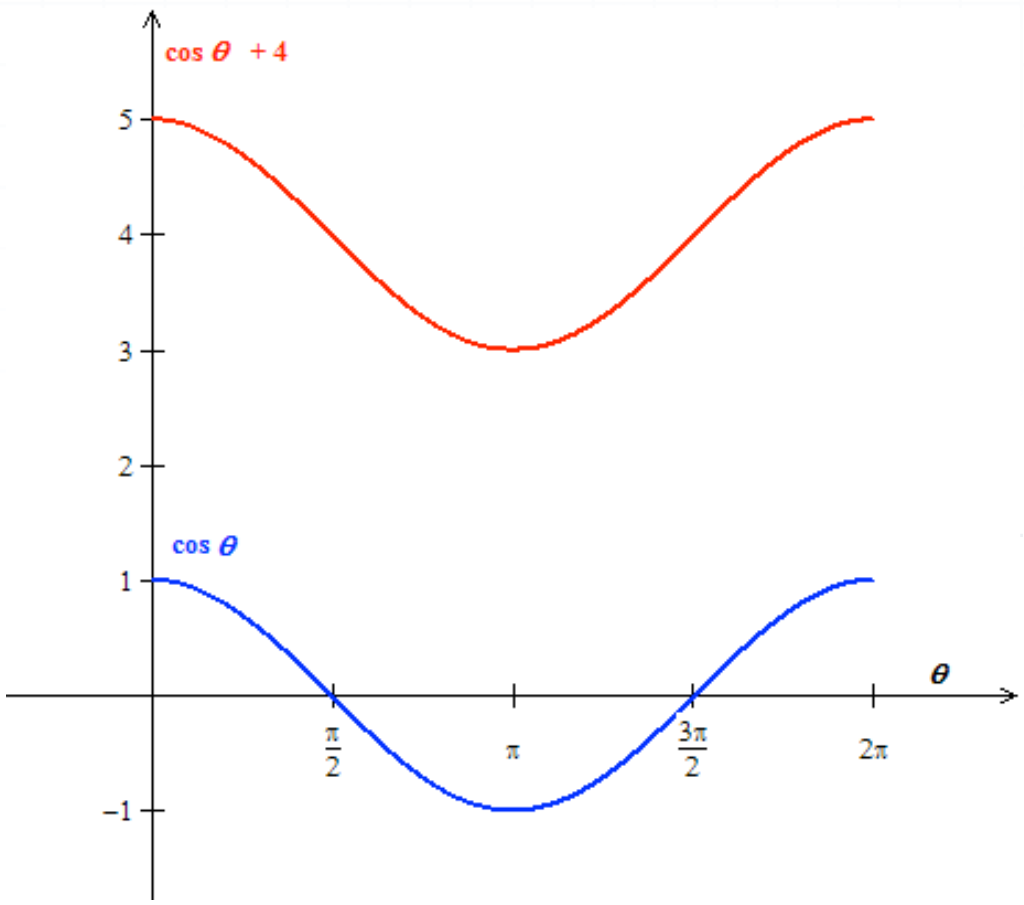
Example

Sketch the graph of $\cos\theta + 4$.

The value of $\cos\theta$ ranges from -1 to 1, so the value of the function $\cos\theta + 4$ ranges from $-1 + 4 = 3$ to $1 + 4 = 5$. In other words, the addition of 4 to the value of $\cos\theta$ shifts the entire graph upward by 4 units, so its value goes from a high of 5 (at $\theta = 0$) down to 4 (at $\theta = \pi/2$) and 3 (at $\theta = \pi$), then back up to 4 (at $\theta = 3\pi/2$) and 5 (at $\theta = 2\pi$).



Let's do another example with the cosine function where $d < 1$.

Example

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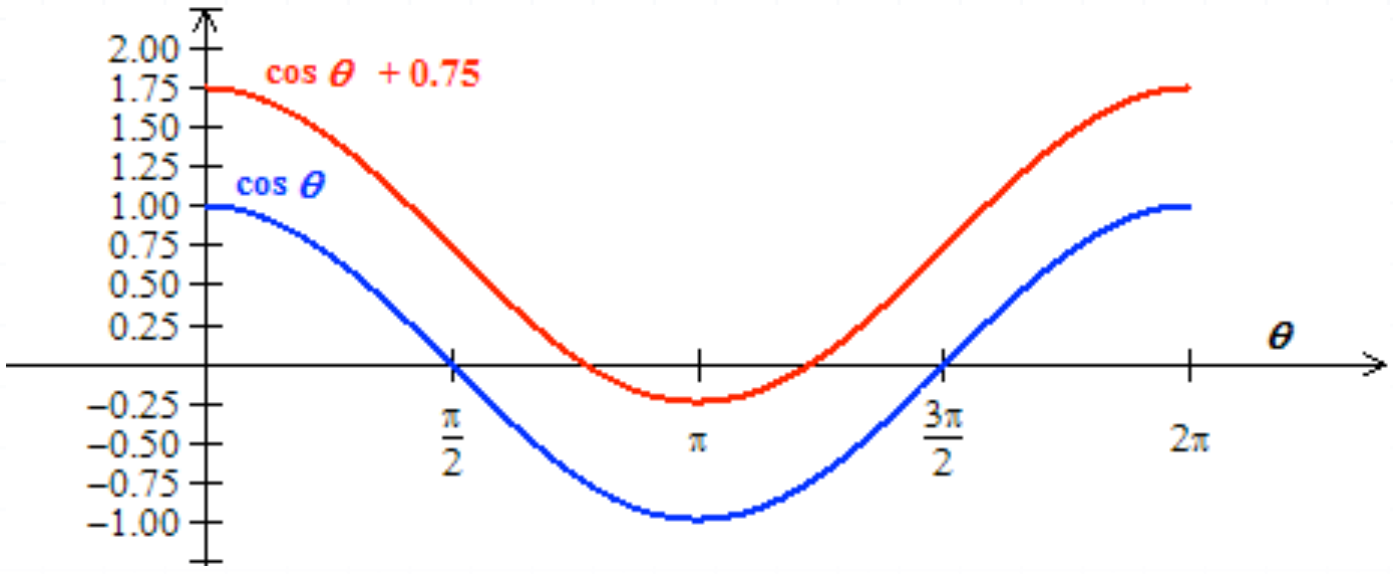
Sketch the graph of $\cos\theta + 0.75$.

The value of this function ranges from $-1 + 0.75 = -0.25$ to $1 + 0.75 = 1.75$. In other words, the addition of 0.75 to the value of $\cos\theta$ shifts the entire graph upward by 0.75 units.



Let’s look at vertical shifts for the sine function.

Example

Sketch the graph of $\sin\theta + 1$.

Let's look at the basic sine function, $\sin\theta$, and consider what the addition of 1 does to it. The value of the basic sine function ranges from -1 to 1, so the value of the function $\sin\theta + 1$ ranges from $-1 + 1 = 0$ to $1 + 1 = 2$.

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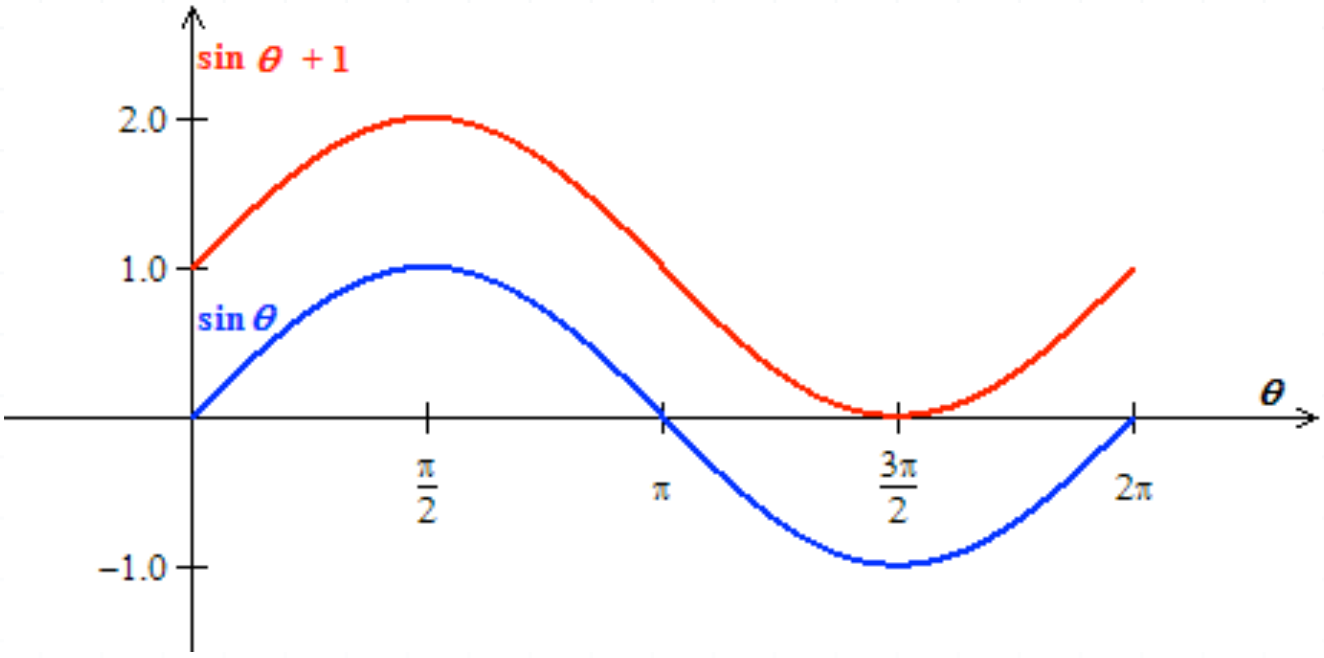
The addition of 1 to the value of $\sin\theta$ shifts the entire graph upward by 1 unit, so its value goes from 1 (at $\theta = 0$) up to 2 (at $\theta = \pi/2$), and then down to 1 (at $\theta = \pi$) and 0 (at $\theta = 3\pi/2$), and back up to 1 (at $\theta = 2\pi$).



Now we'll take a look at a sine function whose graph has a negative vertical shift.

Example

Sketch the graph of $\sin\theta - 3$.

The value of this function ranges from $-1 - 3 = -4$ to $1 - 3 = -2$. The subtraction of 3 from the value of $\sin\theta$ shifts the entire graph downward by 3 units.

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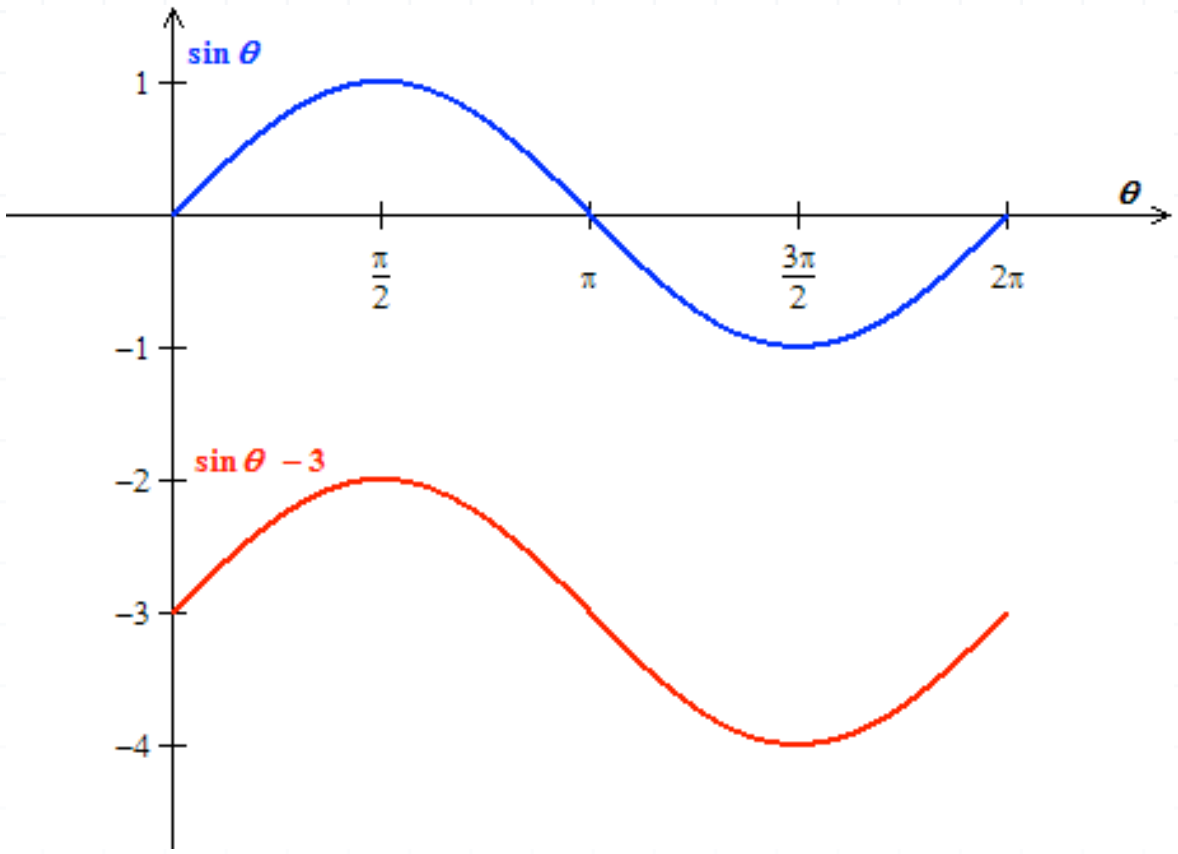

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